GARCH (1,1) as the Stochastic Underlying Process for Stock Market Returns : Empirical Evidence from Asian Markets

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Abstract

Traditional econometric analysis assumes the financial time series as a random walk process with constant variance. However, returns from financial market variables exhibit the conditionally dependent behaviour which is autoregressive in nature and can be explained by the generally auto regressive conditionally heteroskedastic process. The present study focused on GARCH as the process explaining the behaviour for returns and volatility of indices of the stock markets of 10 Asian countries. This process was estimated by employing the GARCH (1,1) model on stock market returns. The time period considered is from January 2000 to July 2013. The GARCH(1,1) model was found to be a satisfactory model fitting the financial time series.

Keywords: **GARCH (1,1), conditional variance, persistence, volatility, ARCH ,volatility clustering**

JEL Classification: G12, G14,G15,G17

The traditional random walk hypothesis states that the stock market prices evolve as a random walk process and thus cannot be predicted. However, the modern auto regressive model describes the financial time series as a stochastic process in which the current value contains information regarding the future value. Auto regressive models capture this time varying behaviour of the financial time series. It states that returns from the financial time series, calculated over high resolution (daily basis) are uncorrelated but not independent. Volatility is found to be time varying and conditional. Financial returns also exhibit the volatility clustering effect, that is, the large returns are followed by larger returns and small returns are followed by smaller returns. Such behaviour is called auto regressive conditional heteroskedasticity (ARCH) effect. In order to capture this behaviour and other stylized characteristics, such as persistence, decay, long memory, and so forth, the generalized ARCH process (GARCH) has been empirically considered as the underlying process for explaining the financial returns (Bollerslev,1986).

The modern framework for time series considers that the volatility is time varying feature of the financial returns considering at the same time that the unconditional covariance remains constant through time. In academic literature, the GARCH(1,1) process is perceived to the realistic data generating process for financial returns (Bera & Higgins, 1993 ; Berkes, Horvath, & Kokoszka, 2003 ; Giraitis, Kokoszka, Leipus, & Teyssiere, 2003). This GARCH process is the main focus of this study.

Review of Literature

Mandelbrot (1963), Fama (1965), and Black (1976) highlighted volatility clustering, leptokurtosis, and leverage

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effect characteristics of stock returns. Engle (1982) introduced the auto regressive conditional heteroskedasticity (ARCH) to model volatility by relating the conditional variance of the disturbance term to the linear combination of the squared disturbances in the recent past. Bollerslev (1986) generalized the ARCH by modeling the conditional variance to depend on its lagged values as well as squared lagged values of disturbances. Since the works of Engle (1982) and Bollerslev (1986), various variants of the GARCH model have been developed to model volatility, for example, EGARCH originally proposed by Nelson (1991), GJR-GARCH model introduced by Glosten, Jagannathan, and Runkle (1993), and the threshold GARCH (TGARCH) model was developed by Zakoian(1994).

Hamilton (1989) employed the GARCH model to model the periodic shifts from recessions to expansions and vice versa of the U.S. business cycle. Hsieh (1989) found that the GARCH (1,1) model worked well to capture most of the stochastic dependencies in the time series. Based on the tests of the standardized squared residuals, he found that the simple GARCH (1, 1) model did better at describing data than a previous ARCH model estimated by Hsieh (1989). Taylor (1994), Brook and Burke (2003), Frimpong and Oteng-Abayie (2006), and Olowe (2009) also found similar results. Lamoureux and Lastrapes (1990) showed that the persistence of financial series might have originated from structural changes in the variance process. Campbell and Hentschel (1992) and Braun, Nelson, and Sunier (1995) provided evidence that stock returns have time varying volatility. Bekaert and Harvey (1997) and Aggarwal, Inclan, and Leal (1999) confirmed the ability of asymmetric GARCH models in capturing asymmetry in stock return volatility for emerging markets volatility. Andersen and Bollerslev (1998) claimed that the GARCH models provide good volatility forecasts. Goyal (2000) estimated that the GARCH volatility frequently lied within the confidence intervals.

Lunde and Hansen (2001) tested whether the evolution of volatility models led to better forecasts of volatility. They did an out of sample comparison of 330 different volatility models (based on conditional variance) using daily exchange rate data and IBM stock prices and found that even the best models did not provide a significantly better forecast than the GARCH(1,1) model. Jaysuriya (2002) examined 15 emerging markets for the effect of stock market liberalization on stock returns volatility for the period from December 1984 to March 2000. He found cyclical behaviour in stock price changes. Starica (2003) investigated how close were the simple endogenous dynamics imposed by a GARCH (1,1) process to the true dynamics of returns of main financial indices. She analyzed the log returns of S&P 500 stock market index from March 4, 1957 to October 9, 2003. She rejected the hypothesis that GARCH (1,1) is true data generating process for longer sample log returns of S&P 500 stock market index. Fryzlewicz (2007) showed that GARCH (1,1) model admitted the unique properties of stationarity, zero mean, lack of serial correlation, heavy tails, and conditional heteroskedastic variance of log returns.

Ashley and Patterson (2010) found that GARCH(1,1) model modeled the daily financial returns of the CRSP equally weighted stock index for the period from January 2006 to December 2007. They compared ARCH, GARCH, and EGARCH models. They found that the GARCH (1,1) model was the only specification for which estimation procedure converged to statistically significant parameter estimates consistent with a stable model. Matei (2009) offered support to the rationale that GARCH is the most appropriate model when one has to evaluate the volatility of the returns of groups of stocks with a large number of observations. He found that in post-estimation part, the GARCH model was a proper model to be used to explain the variances of these indices. Emenike (2010) also employed GARCH (1,1) model to capture the effect of volatility clustering in the Nigerian Stock Exchange from January 1985 to December 2008. They found that volatility of stock returns was persistent in Nigeria. McMillan, Speight, and Apgwilym (2000) analyzed the forecasting performance of a variety of statistical and econometric models of UK FTA All Share and FTSE 100 stock index volatility at the monthly, weekly, and daily frequencies under both symmetric and asymmetric loss functions. They found that the GARCH model provided marginally superior daily volatility forecasts under symmetric loss functions. They concluded that the GARCH models provide, in general, relatively poor volatility forecasts at higher intraday frequency data.

Bonilla and Sepulveda (2011) used the Hinich portmanteau bicorrelation test to detect for the adequacy of using GARCH as the data generating process to model conditional volatility of stock market index rates of returns in 13 emerging economies. They found that GARCH formulation or any of its variant failed to provide an adequate characterization for the underlying process of market indices. Murthy, Anupama, and Deepa (2012) found that the geometric random walk model (ARIMA(0,1,0)) was better than the other ARIMA models over a short-term and long term horizon. Achia, Wangombe, and Anyika (2013) revealed that GARCH (1,1) model provided a better explanation of dynamics of the market returns. Bhanja, Dar, and Samantaraya (2013) employed the Bollerslev's GARCH (1,1) model for measuring the volatility of the nominal and real effective exchange rate for analyzing the impact of exchange rate volatility on India's export growth rate for the time period from April 1993 to September 2010.

Objectives of the Study

The main working hypothesis of this study is to test whether stock market returns follow the GARCH process or not. This can be tested by fitting the GARCH (1,1) model on the stock market returns, and testing whether the GARCH (1,1) explains the financial returns behaviour or not. Secondly, it aims to model the volatility behaviour like volatility clustering, persistence, and decay by considering the GARCH as an underlying process for the time series. Lastly, the study estimates the in-sample forecasting ability of the GARCH (1,1) model.

Research Methodology

The study considers the indices of 10 Asian countries for a long time period - from January 2000 to July 2013. The indices and the country's name have been given in the Appendix 1A. The closing value of the index on a daily basis was taken from the respective websites of the indices. The returns were estimated as the natural logarithms taken as follows, and were used as per the requirement of the test :

$$r_t = \ln (P_t / P_{t-1})$$

First of all, the series are checked for the presence of long run memory by testing it for stationarity or unit root. For testing the presence of unit root, the ADF test was applied.

➯ **Augmented Dickey Fuller (ADF) Test :** For a return series R_t , the ADF test consists of a regression of the first difference of the series against the series lagged k times as follows:

$$\Delta r_t = \alpha + \delta r_{t-1} + \Sigma \beta_s \Delta r_{t-s} + \varepsilon_t$$

where,

$$\Delta r_t = r_t - r_{t-1}$$

The null and alternative hypotheses are $H0: \delta = 0$;and $H1: \delta < 1$. The acceptance of the null hypothesis implies non stationarity. Now, the series are analyzed for the presence of first order autocorrelation.The statistical significance of ACF (auto correlation function) and PACF (partial auto correlation function) can be carried out by testing the joint hypothesis that all ρ_k upto certain lags are simultaneously equal to zero, which can be carried out by Q statistic. The Q statistic is often used as a test of whether a time series is white noise.

The serial autocorrelation is used to test the relationship between the time series and its own values at different lags. If the serial autocorrelation is negative, it means it is mean reverting and the null hypothesis is accepted. If the result is positive coefficients, then it rejects the null hypothesis. Ljung-Box test provides a superior fit to the chi-square distribution. It is defined as $Q = n (n + 2) \Sigma r^2 k/(n - k)$, where n = sample size and k = lag length. After this, the series is checked for the presence of auto regressive conditional heteroskedasticity effect for lag order of one. Its aim is to analyze the presence of volatility clustering and is tested by considering the LM statistic.

↳ **Auto Regressive Conditional Heteroskedasticity- Lagrange Multiplier Test (ARCH-LM Test) :** The ARCH-LM test is a Lagrange multiplier (LM) test which is frequently used to test for the lag length of ARCH errors, in other words, the ARCH-LM test is about testing whether the series has ARCH effects at all (Engle, 1982).
For ARCH-LM test, we run a regression :

$$\varepsilon^2_t = \alpha_0 + (\Sigma^n_{i=1} \alpha_i \varepsilon^2_t \varepsilon^2_{t-n}) + e_t$$

In this regression, $\alpha_0 = \alpha_1 = \alpha_2 = \ldots\ldots\ldots \alpha_n = 0$ is the null hypothesis. Besides, the test statistics follows χ^2 distribution with *n* degrees of freedom. After testing for ARCH effect, we can estimate the generalized autoregressive conditional heteroskedasticity (GARCH) to the logarithmic returns.

↳ **General Auto Regressive Conditional Heteroskedasticity (GARCH) Model :** The GARCH process can be estimated by modeling the GARCH (1,1) model to the log returns. GARCH (1,1) can also be employed for checking the forecasting ability of the returns series. Among the worldwide introduced GARCH family models, the overwhelemingly most popular GARCH model in applications has been the GARCH (1,1) model (Teräsvirta, 2009).The GARCH model is explained as follows:

The return series of a financial asset $\{r_t\}$ is often a serial sequence with zero mean and exhibits volatility clustering. This indicates that the conditional variance or volatility is not constant and is driven by past returns. This implies that we cannot apply the linear regression for modeling the returns or volatility as it considers the returns to be homoskedastic in nature. A standard time series model:

$$r_t = E(r_t | \Omega_{t-1}) = \varepsilon_t$$

According to Bollerslev (1986), r_t denotes the real-valued discrete time process with conditional mean and variance, which vary with Ω_{t-1}, where Ω_{t-1} is the information set of all information through time *t*. In this study, r_t is the returns which are equal to logarithmic returns of the financial time series. It is common to assume that logarithmic returns are normally distributed on all time resolutions, whether daily, weekly, or yearly. If the log returns are normally distributed, then the prices would never go negative, which is economically not possible.

An auto regressive moving average (ARMA) *(p,q)* model has the mean equation :

$$E(r_t | \Omega_{t-1}) = \mu(\theta)$$
$$\mu(\theta) = \varphi_0 + \varphi_1 r_t + \cdots\ldots.. + \varphi_p r_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_p \varepsilon_{t-q}$$

ARMA model is commonly used as a mean equation for the return series. Then, the ARCH model (Engle,1982) can be treated as the variance function, that is,

$$Var\, E(r_t | \Omega_{t-1}) - E(\varepsilon^2_t | \Omega_{t-1}) = h_t(\theta)$$

The ARCH *(q)* process function is given below:

$$h_t - \omega + \alpha_1 \varepsilon^2_{t-1} + \cdots.. + \alpha_q \varepsilon^2_{t-q}$$

Bollerslev (1986) recognized the difference between unconditional and conditional variance, allowing the latter to change over time as a function of past errors. The ARCH model has been replaced by the generalized ARCH (GARCH) model given by Bollerslev (1986) .The GARCH *(p, q)* process is given as :

$$r_t = E(r_t | \Omega_{t-1}) + \varepsilon_t$$
$$\varepsilon_t | \Omega_{t-1} \sim N(0, h_t)$$
$$h_t - \omega + \alpha_1 \varepsilon^2_{t-1} + \cdots + \alpha_q \varepsilon^2_{t-q} + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-p}$$

The conditional distribution of ε_t is supposed to be normally distributed with zero mean and the conditional variance equal to h_t.

$\Sigma^p_{i=1} + \beta_i h_{t-i}$ is the GARCH term. For $p = 0$, the process is simply ARCH *(q)* process; for $p = q = 0$, ε_t is only white noise.

The simplest GARCH (1,1) model is, thus, given below:

$$r_t = E(r_t | \Omega_{t-1}) + \varepsilon_t$$
$$\varepsilon_t | \Omega_{t-1} \sim N(0, h_t)$$
$$h_t = \omega + \alpha \varepsilon^2_{t-1} + \beta_1 h_{t-1}$$

where,

$\omega > 0, \alpha > 0, \beta > 0$

The size of α and β determine the short run dynamics of the resulting volatility time series in terms of persistence and reaction of stock returns to the market movements and shocks. In GARCH (1,1) model, the effect of return shock on current volatility declines geometrically over time.

Diagnostics of GARCH (1,1) Model

- **Log Likelihood Ratio:** The log likelihood ratio constitutes one of the best ways to measure and express the diagnostic accuracy of a model. It is based on the maximum likelihood principle. Under the assumption that disturbances ε_i are normally distributed, the maximum likelihood estimators of the regression coefficients are identical, but the estimated error variances are different. The LR test obtains the statistic as follows $\lambda = 2 \ln$ (ULLF- RLLF), where ULLF is unrestricted log likelihood function and RLLF is restricted log likelihood function.

- **AIC:** The Akaike information criterion (AIC) measures the relative goodness of fitting statistical models. The values of AIC calculated in this study are based on maximizing the log likelihood function (normal errors).The AIC is calculated as below:

$$\text{AIC} = \ln\ (\text{SSE}/n) + 2K/n$$

where,

n is the number of observations and SSE is squared standardized errors.

- **BIC :** The Bayesian information criterion or Schwartz criteria is also based on likelihood function and it is quite close to AIC. The Schwartz criteria is based on the assumption that the model errors or disturbances are independently and identically distributed according to normal distribution. BIC is calculated as below :

$$\text{BIC} = \text{SC} = \ln(\text{SSE}/n) + \text{K} \ln(n)/n,$$

where,

n is the number of observations and SSE is the squared standardized errors.

- **HQC:** Hannan Quinn information criteria is an information criterion for model selection. It is calculated as below:

$$\text{HQC} = n \log(\text{RSS}/n) + 2k \log \log n$$

where,

k is the number of parameters, n is the number of observations, and RSS is the residual sum of squares that results from linear regression or other statistical model.

- **Performance Measures :** The performance measures which are used to evaluate the forecast errors in the volatility forecasting are mean error, mean squared error, root mean squared error, and mean absolute error.

The mean error *(ME)* of in sample estimates $(\hat{r})$ relative to actual values *(r)* can be defined as $ME = \overline{(\hat{r}) - (r)}$ where, the over-bar denotes an average over a large sample in time. A perfect score of zero does not exclude very large errors of opposite signs which cancel each other out.

The mean squared error (MSE) is the risk loss function with quadratic loss function. It measures the average of squared difference between the estimated return $(\hat{r})$ and the actual value (r), and can be defined as $MSE = ME = ((\hat{r}) - (r))2$.

The root mean square error *(RMSE)* is the quadratic scoring rule which measures the average magnitude of the error, and it is calculated as $RMSE = \sqrt{\overline{(\hat{r} - r)^2}}$. It gives relatively high weight to a large error. It is more useful when the large errors are particularly undesirable.

Results and Discussion

The Table 1 showcases the descriptive statistics for the log returns of the daily closing prices of the considered indices. Almost all the indices have positive mean return, indicating that the returns have increased over the period considered. Only Japan and Taiwan have negative average returns, indicating a decrease in the return over the period considered. The standard deviation and variance of the indices are small.

Skewness for all the indices is negative, implying that the return distributions of indices have a higher probability of earning negative returns. Kurtosis of all the indices is greater than three indicating heavier tails and non normal distribution, which is also confirmed by the Jarque Bera test for normality (shown in the Table 2). The null hypothesis for the Jarque Bera test is that the 'Series is normal,' and it is rejected in all the cases.

The Table 3 contains the results of the ADF test. Augmented Dickey Fuller (ADF) test was conducted to test the presence of unit root in the financial time series. It confirms whether the series is random or is stationary in nature. The null hypothesis for the ADF test is that the series contains unit root and is not stationary. The *p* - value

Table 1. Descriptive Statistics of Logarithmic Returns for Indices of Different Countries

Particulars	Australia	Hong Kong	India	Indonesia	Japan	Korea	Malaysia	Singapore	Taiwan	China
Mean	0.00014	0.00007	0.00038	0.00057	-0.00010	0.00018	0.00023	0.00006	-0.00002	0.00011
Standard Error	0.00017	0.00027	0.00028	0.00026	0.00027	0.00030	0.00019	0.00021	0.00026	0.00027
Standard Deviation	0.01005	0.01597	0.01632	0.01476	0.01584	0.01720	0.01123	0.01240	0.01522	0.01577
Variance	0.00010	0.00025	0.00027	0.00022	0.00025	0.00030	0.00013	0.00015	0.00023	0.00025
Kurtosis	6.34587	7.69089	6.46212	6.06602	6.33338	5.15596	90.94391	6.02086	2.65062	4.53928
Skewness	-0.60587	-0.06558	-0.18016	-0.69029	-0.42126	-0.52424	-0.24265	-0.42411	-0.23341	-0.09360
Range	0.13914	0.26989	0.27799	0.18577	0.25346	0.24089	0.39107	0.16746	0.16461	0.18657
Minimum	-0.08554	-0.13582	-0.11809	-0.10954	-0.12111	-0.12805	-0.19246	-0.09216	-0.09936	-0.09256
Maximum	0.05360	0.13407	0.15990	0.07623	0.13235	0.11284	0.19860	0.07531	0.06525	0.09401
N	3440	3392	3363	3285	3332	3352	3345	3423	3347	3452
Confidence Level (95.0%)	0.00034	0.00054	0.00055	0.00050	0.00054	0.00058	0.00038	0.00042	0.00052	0.00053

Table 2. Results of the Jarque-Bera Test

	Australia	Hong Kong	India	Indonesia	Japan	Korea	Malaysia	Singapore	Taiwan	China
Jarque Bera statistic	5962.39	8333.8	5849.04	5278.84	5647.44	3852.65	1.15E+06	5254.71	1005.92	2957.9
p - value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 3. Results of the Augmented Dickey Fuller (ADF) Test

Index	ADF test with constant	ADF test with constant and trend	ADF test with constant and trend squared
Australia	-10.1717	-10.1702	-10.1691
	9.32E-20	1.74E-20	1.33E-92
Hong Kong	-10.0049	-10.0212	-10.093
	3.20E-19	6.99E-20	1.97E-87
India	-10.5451	-10.5608	-10.774
	5.79E-21	4.23E-22	8.40E-146
Indonesia	-9.212	-9.22799	-9.35011
	1.04E-16	8.66E-17	1.06E-49
Japan	-10.4333	-10.5218	-10.5219
	1.33E-20	6.16E-22	1.15E-120
Korea	-9.94998	-9.95375	-10.0537
	4.80E-19	1.31E-19	7.19E-85
Malaysia	-10.212	-10.3099	-13.7292
	6.91E-20	4.67E-21	0.00E+00
Singapore	-9.78622	-9.80084	-9.8427
	1.60E-18	5.30E-19	1.87E-72
Taiwan	-9.84072	-9.8981	-9.99955
	1.07E-18	2.18E-19	1.86E-81
China	-12.5826	-12.5917	-12.615
	1.33E-27	3.71E-31	0.00E+00

Table 4. Results of Ljung Box Test (*Q* Statistic) and Engle's ARCH Test (LM statistic) for the Lag Order 1

Index	*Q* - statistic	*p* - value	LM statistic	*p* - value
Australia	2.2002	0.138	215.895	0.0000
Hong Kong	0.8228	0.364	402.332	0.0000
India	16.8759	0.000	144.452	0.0000
Indonesia	39.555	0.000	103.075	0.0000
Japan	4.1994	0.040	244.772	0.0000
Korea	1.2039	0.273	89.8171	0.0000
Malaysia	28.805	0.000	764.748	0.0000
Singapore	0.2791	0.597	83.2856	0.0000
Taiwan	6.7316	0.009	66.157	0.0000
China	0.0688	0.793	62.1768	0.0000

indicates the rejection of the null hypothesis, thereby confirming that the return series of the indices are stationary in nature. The ADF test was conducted on the lag length of 28. The series was then tested for the serial correlation for the order of one in order to estimate the mean reversion of the series. It is tested with the help of Ljung Box Q statistic for order of one. It estimates the relationship between the present value of the series with the lagged value of the same series. It basically tests the randomness of the sample. The null hypothesis for the Ljung Box statistic is that the series is independently distributed.

The results for the Q - statistic are shown in the Table 4. It can be observed that the null hypothesis is rejected for half of the indices, and is accepted for the rest. Considering the results of the ADF test and the Ljung Box Q statistic together, it can be observed that there is a presence of first-order serial correlation, which actually infers

Table 5. Results of the Estimated Parameters and Diagnostics of the GARCH (1,1) Model

Index	constant	ω	α	Β	Log likelihood Ratio	Schwarz Criteria	Akaike Information Criteria	Hannan Quinine information Criteria	Un-conditional variance
Australia	0.00058	0.00000	0.10393	0.88747	11627.37	-23214.03	-23244.75	-23233.78	0.00012
	0.00012	0.00000	0.01631	0.01575					
Hong Kong	0.00046	0.00000	0.06267	0.93060	9859.68	-19678.71	-19709.36	-19698.40	0.00022
	0.00020	0.00000	0.00837	0.00836					
India	0.00096	0.00000	0.11816	0.86685	9625.74	-19210.88	-19241.48	-19230.54	0.00030
	0.00021	0.00000	0.02009	0.02302					
Indonesia	0.00118	0.00001	0.13759	0.83026	9548.56	-19056.64	-19087.13	-19076.21	0.00026
	0.00021	0.00000	0.02600	0.0319					
Japan	0.00040	0.00000	0.10376	0.88238	9513.69	-18986.82	-19017.37	-19006.44	0.00029
	0.00021	0.00000	0.01494	0.01459					
Korea	0.00078	0.00000	0.08141	0.91450	9406.80	-18773.02	-18803.61	-18792.67	0.00046
	0.00022	0.00000	0.01308	0.01235					
Malaysia	0.00012	0.00001	0.08723	0.82159	10805.57	-21570.56	-21601.14	-21590.20	0.00011
	0.00018	0.00000	0.0337	0.04547					
Singapore	0.00051	0.00000	0.09598	0.90161	10759.02	-21477.34	-21508.03	-21497.07	0.00045
	0.00015	0.00000	0.01664	0.01475					
Taiwan	0.00049	0.00000	0.06672	0.92679	9688.42	-19336.25	-19366.83	-19355.89	0.00026
	0.00020	0.00000	0.01192	0.01244					
China	0.00015	0.00000	0.06185	0.92685	9788.65	-19536.57	-19567.30	-19556.33	0.00026
	0.00022	0.00000	0.01485	0.01785					

Table 6. Results for the Ljung Box *Q* Statistic for Squared Standardized Residuals of GARCH (1,1) Model

	Australia	Hong Kong	India	Indonesia	Japan	Korea	Malaysia	Singapore	Taiwan	China
Q- statistic	0.117	5.447	0.2804	0.0182	0.1346	1.9319	1.5443	2.2082	2.9947	0.0006
p - value	0.732	0.02	0.596	0.893	0.714	0.165	0.214	0.137	0.084	0.981

that the return of yesterday contains the information for the return of today. This confirms the presence of stylized fact of long memory over the period in the series considered. As the stylized fact of long memory in the series has been indicated, thus the series must be tested for the presence of volatility clustering in the series. This can be looked into by testing for the ARCH effect for the lag order of one, with the help of LM statistic. The results for the ARCH test are given in the Table 4. The null hypothesis considered is that 'There is no ARCH effect present'. It can be interpreted that the null hypothesis is rejected for all the series, thereby confirming the presence of ARCH effect in the series. This indicates that the GARCH (1,1) model for modeling the returns and volatility can be applied to all the considered series.

The results for the GARCH (1,1) model for estimating the variance of the time series of log returns of the indices are given in the Table 5. The diagnostic test for the GARCH (1,1) model is how well the model fits the data. Firstly, if the model is able to describe the data, then the standardized residuals should be independently and identically distributed. Secondly, if the volatility clustering is explained by the model, then the squared residuals should have zero autocorrelation. Both can be tested by applying the Ljung Box *Q* statistic on the standardized squared residuals obtained out of the model applied.

The results for the *Q* statistic are shown in the Table 6. The null hypothesis of no autocorrelation for lag order of

Table 7. Results for ME, MSE, RMSE, and MAE for the In- Sample Forecasting Estimates

Index	Mean Error	Mean Squared Error	Root Mean Squared Error	Mean Absolute Error
Australia	-0.00045	0.000101	0.010062	0.006981
Hong Kong	-0.00039	0.000255	0.015969	0.010906
India	-0.00058	0.000267	0.016326	0.011469
Indonesia	-0.0006	0.000218	0.014771	0.0103
Japan	-0.0005	0.000251	0.015842	0.011336
Korea	-0.00061	0.000296	0.017203	0.011956
Malaysia	0.000105	0.000126	1.12E-02	0.006325
Singapore	-0.00045	0.000154	0.012404	0.0086
Taiwan	-0.00052	0.000232	1.52E-02	1.08E-02
China	-3.60E-05	0.000249	0.015771	0.010731

one is accepted in case of all the indices. As the GARCH (1,1) model fits the data, the results for the parameters obtained for modeling the volatility can be interpreted now, and the model can be used to forecast volatility as well. The log likelihood value considered along with information criteria like Schwarz information criteria (SIC), Akaike information criteria (AIC), Hannan Quinine information criteria (HQIC) indicates the relative goodness of fitting the model statistically. In all the cases, SIC, AIC, HQIC have high negative values supported by high positive log likelihood values, indicating the reliable estimation of the parameters. The short run dynamics and persistence of volatility are reflected in the values of parameters α and β. As shown in the Table 5, it can be observed that the values of the β are close to one or high, which indicates that the shocks to conditional variance take a long time to die out. This indicates the persistence in the volatility. The comparatively smaller values of α indicate that the large market surprises induce relatively small reversions in future volatility. Furthermore, it can be observed that the sum of α and β is approximately equal to 1 in all the cases. It indicates that a shock at time *'t'* will persist for many future periods and there is long memory in the variance of the market returns. In other words, it can be said that volatility decays itself over the period of time. As the time frame considered is long term, so the decay factor of volatility can be trusted upon with reliability. It also indicates that today's variance contains the information for tomorrow's variance, which is more evident in case of Indian and Australian markets. This fact suggests that the returns can be forecasted by employing the GARCH (1,1) model.

The goodness of fit for GARCH model is estimated by checking the significance of parameter estimates and how well it models the volatility of the series. If the model adequately captures the volatility clustering, then the standardized squared residuals should have no autocorrelation, which can be tested with the help of Ljung Box Q statistic. The forecasting ability of GARCH (1,1) model is estimated by employing the ME, MSE, RMSE, MAE. The results in the Table 7 show that the mean error for all the indices is approximately equal to zero. The mean error cancels out the errors of opposite signs, so it is better to consider the mean squared error along with it. The MSE is a risk function and incorporates both the variance of the estimator and its bias. An ideal value equal to zero of MSE shows the estimator's ability to provide forecasts with perfect accuracy. It can be observed that the value of MSE is almost zero, indicating the better fit of the model for forecasting. Mean absolute error and the root mean squared error have linear and quadratic scores respectively.

They can range from zero to infinity. MAE gives equal weights to all errors, and RMSE gives high weight to large errors. As shown in the Table 7, both have small values, which is a favourable indication for better fit of the model. Considering RMSE and MAE together to diagnose the variation in the errors in a set of forecasts, it can be observed that the difference between RMSE and MAE is very small. This indicates small variance in the individual errors in the sample.

Conclusion and Implications

The present research paper has explored how the Asian stock markets behave, given the GARCH framework. We adopted the GARCH (1,1) model as the underlying process for the long term of 13 years. The results reveal that the in sample forecasting ability of GARCH (1,1) model for the variance and returns is satisfactory. The results show that the Indian and Australian markets have comparatively longer persistence as compared to others, especially the Chinese market.

The research findings on the stochastic nature of indices reveal that the Asian markets are volatile in nature, and the volatility has persistence. The persistence and memory of stock markets have tremendously constrained the investment activities. Therefore, these research findings have far reaching implications on the financial modeling of returns and portfolio management. As we have considered a longer term for the analysis, so it is assumed that the effects of jumps as separate events could be ignored. Therefore, an important extension of our work would be to empirically examine how these indices behave with the inclusion of the structural breaks.

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Appendix 1A

List of indices and the stock markets considered for the study

Name of the Index	Country	Stock Market
ALL ORDINARIES	Australia	Australian Stock Exchange
HANG SENG Index	Hong Kong	Hongkong Stock Exchange
COMPOSITE Index	Indonesia	Indonesia (formerly known as Jakarta) stock exchange
FTSE Bursa Malaysia KLCI Index	Malaysia	Malaysia Stock Market
NIKKEI 225 Index	Japan	Tokyo Stock Exchange
KOSPI Composite Index	Seoul	Korean Stock Exchange
TSEC weighted index	Taiwan	Taiwan Stock Exchange
SSE Composite Index	China	Shanghai Stock Exchange
BSE 30 Index	India	Bombay Stock Exchange