Long Memory in Indian Equity Market : De-trended Fluctuation Analysis

** Saji George*
*** P. Srinivasa Suresh*

Abstract

The presence of diverse investor categories of varying sophistication and the role of wealth funds along with other characteristics has left the Indian equity market susceptible to inefficiencies in the price discovery process. This study examined the long memory possibilities in the Indian market across various indices representing market coverage, market capitalization, and liquidity based portfolios, both in the pre and post financial crisis period of 2008. The null hypothesis that prices follow a pure random process was tested against the long-range dependence. The results based on de-trended fluctuation analysis technique indicated relatively weaker long range dependence in all the indices and higher level of persistence in their volatility. The study also observed varying degrees of persistence across the market cap and liquidity based portfolio indices and the persistence was found to be higher in the post-crisis period. Broadly, the nature of the dependence was found to be different both in price indices and in their volatility. Across market cap portfolios, the temporal dependence was found to be negatively associated, that is, large cap and medium cap security price indices movements evinced lower long range dependence compared to that with small capitalization portfolio price indices. Contrary to this, long range dependency in their volatility was found to be increasing with the market size. Linkage between level of persistence and market liquidity price indices was found to be weak. NSE indices evinced more efficiency than BSE indices. In short, price formation in Indian market evinced long memory, and this information was decisive, especially in pricing of index linked derivatives and other funds traded in the Indian market. It also indicated the relevance of non-linear asset pricing models in the Indian equity market.

Keywords: **detrended fluctuation analysis, long memory, market capitalization and liquidity, market returns and volatility**

JEL Classification: C10, G12, G14, G19

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Theoretically, as argued by the proponents of efficient market hypothesis, the equity price formation is expected to follow pure random process in a market wherein information instantaneously gets incorporated in prices by the trading activities of rational traders and arbitrageurs. However, this argument was countered by arguments such as trade impossibility in efficient markets by Marshal (1974); the possibility of bias in the posterior decisions even in the presence of equal priors due to psychological factors by Aumann (1976). The empirical evidences of aggressive information seeking activities, emergence of advisory firms in trading related services, evidences of correlated price generating process from various markets, and profitable trading strategies have challenged the arguments of informationally efficient price formation process. At present, the debate in the financial market literature has gone further on to the nature of dependencies in the

* *Ph.D. Scholar*, Department of Economics, North Eastern Hill University, Shillong - 793 022, Meghalaya.
E-mail : sjtjohn2008@yahoo.co.in
** *Associate Professor,* Department of Economics, North Eastern Hill University, Shillong - 793 022, Meghalaya.
E-mail : sureshps03@yahoo.co.in

price formation process. The enquiry of long memory in equity prices or returns, therefore, is a verification of market efficiency. Technically, time series data often portrays two types of temporal behaviour based on the nature of dependence in their increments, short range dependence, and long memory. Short range dependence can be defined as the temporary deviation of prices from long terms mean which is termed as a strong mixing behaviour, wherein maximal dependence between price increments at two dates become trivially small as the time lags expand at a faster rate. On the other hand, in cases where the dependence in the price process decreases very slowly as the time lag expands or if power law decay in time (Joseph Effect) is followed, it would be affected by infinite variance syndrome. This characteristic of the time series data with higher order correlation structure is described as long range dependency or long memory. The presence of long memory can be attributed to long term multiplier effects of transitory shocks and slow information absorption capability of the market which results in full arbitraging impossible in the market contradicting to arbitrage based market correction arguments. Moreover, long range dependence in the market determined prices affects the portfolio management decisions sensitive to investment horizon and also the pricing of derivative securities with Martingale models as time stochastic process employed would become inconsistent. It creates nonlinear dependency in the form of hyperbolically decaying autocorrelations and low frequency spectral distributions, and generates a potentially predictable component in the price data (Lo, 1991).

Most of the studies carried out across various markets, including the Indian market, except few earlier period studies, have identified a higher degree of long range dependence in the formation process of equity returns and in their volatility, especially in emerging economies compared to that of developed economies. This persistence is found to be varying across time as well. These studies have also observed evidences of changes in long range dependences together with the transformation of the respective markets. Therefore, this study attempts to re-examine the presence of long memory in Indian equity market returns, volatility in both pre and post crisis period, and also across the market cap and liquidity based portfolio returns and volatility. The study draws its significance as it considers different phases of integration of Indian market with global markets, periods of various crises the economy passed through, various stages of technological upgradation in the trading mechanism, as well as information dissemination processes, large scale entry and exit of different investor categories. This study also attempts to examine the price formation process across market capitalization and liquidity based portfolios.

Long Memory in the Stock Market Returns

The long memory analysis has been carried out in financial markets on various dimensions across economies and markets. This section briefly gives an account of the empirical evidences from markets in Asia, Asia-Pacific, Europe, Africa, North-America, and Latin American regions. Chow, Pan, and Sakano (1996) in an examination of 22 markets such as United States, Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, India, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Philippines, South Africa, Spain, Sweden, Switzerland, and United Kingdom broadly from 1962 to 1990 observed short term dependence in case of Denmark, Finland, Italy, Japan, New Zealand, and South Africa and no evidence of long range dependence in any of these markets.

Matteo, Aste, and Dacorogna (2005) examined the market efficiency in terms of the scaling exponents of number of markets at different stages of development; mature, liquid markets, emerging, and less liquid markets. The study also covered data on daily values of equity indices of 32 markets - U.S., Japan, France, Australia, UK, Netherlands, Germany, Switzerland, New Zealand, Israel, South Korea, Canada, Italy, Spain, Taiwan, Argentina, Hong Kong, India, Brazil, Mexico, Singapore, Hungary, Poland, Malaysia, Thailand, Philippines, Venezuela, Peru, Indonesia, and Russia were considered in the analysis. The study observed significant long range dependence in case of emerging markets and short memory process in the indices of developed markets. Podobnik, Fu, Jagric, Grosse, and Stanley (2006) also observed strong and medium long range dependence in

European transition economies such as the Czech Republic, Hungary, Russia, Slovenia, Croatia, Lithuania, Estonia, and Latvia, while Poland and Slovakia showed weak long range auto - correlations.

The North African markets such as Tunisia, Egypt, and Morocco evinced significant short memory behaviour in the daily close prices of stocks and in their volatility (Onour, 2010). Bhattacharya and Bhattacharya (2012) investigated the existence of long memory in the daily closing prices of market indices of 10 emerging economies classified by Morgan Stanley Capital International such as Hungary, China, Brazil, Chile, Malaysia, South Korea, Russia, Mexico, India, and Taiwan over the period from 2005 to 2011. Short range dependence in logarithmic returns and significant long memory in absolute and squared returns were observed in this analysis. Sensoy (2013) considered daily closing prices of 19 FEAS (Federation of Euro Asian Stock Exchanges) countries and examined the time varying long range dependence in these markets. The markets covered are from the economies of Bahrain, Bosnia, Herzegovina, Bulgaria, Croatia, Egypt, Georgia, Iran, Jordan, Kazakhstan, Macedonia, Mongolia, Montenegro, Oman, Pakistan, Palestine, Romania, Serbia, Turkey, and United Arab Emirates. The average value of time varying Hurst exponent was found to be above 0.5, indicating the persistence of long memory in all these market indices. A tendency towards efficiency was observed among Eastern European markets over the period while for the Middle East market, the divergence was increasing. Least long range dependency or higher market efficiency was observed in Turkish and Romanian markets while Iran, Mongolia, Serbia, and Macedonia were found to be least efficient or having the highest long range dependence.

There are a number of investigations on the level of persistence of prices discovered in the Indian equity market. Banerjee and Mulligan (2010) ; Bhattacharya and Bhattacharya (2012) ; Biswal and Kamaiah (2011) ; Hiremath and Kamaiah (2010) ; Hiremath and Kumari (2015) ; Kumar and Maheswaran (2013) ; Maheshchandra (2014) ; Mukherjee, Sen, and Sarkar (2011) ; Parthasarathy (2013) ; and Nath and Reddy (2003) are the noted studies in this dimension on the Indian market. A deviation from power law to exponential distribution (memory less distribution) was detected in NSE Nifty intraday return movement and daily closing data on the same of Nifty and Sensex (Pan & Sinha, 2007). Mishra, Seghal, and Bhanumurthy (2011) observed return series having the property of fat tail distribution, rejection of random walk hypothesis, and non linear dependency of stochastic nature.

Statistical Techniques of Long Memory Analysis

In this section, an account of statistical technique for estimating the long range dependence is presented. One of the important aspects of long range dependence is that the dependence in auto-covariance of the process follows power law decay in time, that is, its autocorrelation function displays a persistence that is inconsistent with $I(0)$ and $I(1)$ integrated processes. The order of integration $I(d)$ would take the value $0 < d < 0.5$ in case of long range dependent process, and all its autocorrelations would be positive and decay at a hyperbolic rate as in the fractional Brownian process, which is deemed as approximate fractional derivative of a regular Brownian process (Baillie, 1996). It is found in literature that the long memory is examined based on the estimated Hurst exponent value (Hurst, 1951) from the data. The fractional differencing parameter d is related to H exponent as $d = H - 1/2$. Theoretically, when Hurst exponent *(H)* H is 0.5, the fractional Brownian motion reduces to Brownian motion. It indicates two possible processes, independent process and short range dependent process (a process with quickly declining autocorrelations and high frequency spectral distributions). $H > 0.5$ indicates significant positive correlation or persistence in data, while $H < 0.5$ indicates anti-persistence or mean reversion. The long memory analysis focuses on examining if $H > 0.5$. Such a process will have slower decaying autocorrelations and low frequency spectral distributions.

There are various methods to estimate the H exponent. The pioneer and widely used method in earlier studies is rescaled range analysis (R/S statistic) of Hurst (1951) and its modified version by Mandelbrot (1972). However, the evidences observed based on this method were under apprehension as Lo (1991), Chow et al., (1996), and

Krištoufek (2010) observed inefficiencies of R/S statistic in certain conditions. These papers observed that the R/S statistic is incompatible to examine long memory as it is incapable of distinguishing short range dependence from long range one in the series. The statistic is sensitive to short term dependence and it overestimates H exponent in small samples as well as in shorter time scales, which leads to wrong conclusions.

In order to circumvent this problem, Lo (1991) proposed a modified R/S statistic which is invariant over a general class of short memory process and deviates for long memory process. However, Teverovsky, Taqqu, and Wilinger (1999), based on Monte Carlo estimation, pointed out that the modified R/S statistic has a strong bias towards accepting the null hypothesis as the lag increases since it takes into account the covariance in first q lags considered. The appropriate lag selection becomes a major problem, that is, if smaller lags are selected, it may not account for short range dependence, while larger lag selection can overcompensate and eliminate long memory in the process.

Due to the fractional integration process involved, the autoregressive moving average (ARMA) models were found to be inefficient in detecting the long term persistence as these models imposed exponential rate of decay in autocorrelations. Granger and Joyeux (1980) proposed auto-regressive fractionally integrated moving average (ARFIMA or FARIMA) models to examine this feature of the process. It captures the slowly decaying auto-covariance function of a long memory process by letting fractional differencing parameter to take non-integer values in the models. These models are estimated by parametric, semi-parametric, and wavelet methods. An advantage of ARFIMA *(p, d, q)* model is in its ability to simultaneously and separately model short memory (by combining p and q parameters) and long memory based on the parameter d. However, ARFIMA models are extensions of ARIMA *(p, d, q)* models and are linear time series models. Since long memory time series have both linear and nonlinear structures and focus on the differencing parameter d, ARFIMA models may be inadequate to examine long memory of such time series wherein these characteristics are present. Also, in the cases of time series with frequent structural breaks, the efficiency of ARFIMA models were questioned as this model requires large sample of data and possible prevalence of spurious long memory detection (Aladag, Egrioglu, & Kadilar, 2012 ; Gabriel & Martins, 2004 ; Graves, Gramacy, Watkins, & Franzke, 2014 ; Song & Bondon, 2011).

Similar to the ARFIMA or FARIMA models which examine long run dependences in conditional mean of time series, considering the observation of temporal volatility in absolute and squared returns in stock market, Baillie (1996) proposed fractionally integrated generalized autoregressive conditional heteroscedasticity (FIGARCH) models that can capture slow hyperbolic rate of decay for the lagged squared innovation in conditional variances. In these models, although the process is mean-reverting, the shocks to conditional variance die out at a slow hyperbolic rate determined by the fractional differencing parameter, while short term dependencies are modelled by GARCH parameters. But the standard FIGARCH model applied in long memory studies does not account for the possible structural breaks and the resultant chance of detecting spurious long memory in volatility (Baillie & Morana, 2009 ; Wang, Liu & Gu, 2009).

Another widely applied technique in long memory analysis is de-trended fluctuation analysis (DFA) ; a method initially proposed by Peng, Buldyrey, Havlin, Simon, Stanley, and Goldberger (1994) that underwent various modifications. This method enables to determine the mono-fractal scaling behaviour of a noisy data in the presence of trends even without knowing their origin and shape and to detect long range correlations in noisy non-stationary time series by systematically eliminating inherent trend across all time scales and by avoiding the spurious detection of correlations that are artefact of non-stationarity in such data. Kantelhardt, Koscielny - Bunde, Rego, Havlin, and Bunde (2001) observed that the traditional DFA evincing different fluctuation functions for different de-trending orders at small time scales could lead to overestimation of the fluctuation exponent and they proposed a modification in the traditional DFA fluctuation function which was applicable in DFA of any order. With the observation of inefficiencies observed in the static high pass trend filtration applied in the standard DFA process, several studies have come out with variants of DFA technique using different trend filtration processes such as central moving average (CMA) (Ramirez, Rodriguez, & Echaverria, 2005), backward moving average (BMA) (Alessio, Carbone, Castelli, & Frappietro, 2002), recursive high pass filters, etc. The standard DFA is basically a mono-fractal analysis, but when the significant crossover exits either between smaller scales to larger scales or separately for each scale, multitude of scaling exponents are required for the full description of the scaling behaviour in the series.

Data and Methodology

This study examined long memory in the Indian equity market across benchmark and broad market indices, market cap based portfolio indices, liquidity based portfolio indices, and in their volatility. The analysis was carried out across the entire data period available and across two sub periods, and between pre and post global financial crisis period of 2008 as well. Broad market indices such BSE 500 (January 2, 2000 - October 31, 2016), NIFTY 500 (June 7, 1999 - October 31, 2016) ; bench market indices such as BSE SENSEX (January 2, 1992- October 31, 2016) and NIFTY 50 (January 2, 1995 - October 31, 2016) were considered in the first category. In case of market cap based indices, market returns on BSE Large Cap (September 16, 2005 - October 31, 2016), BSE Mid Cap (April 3, 2003 - October 31, 2016), BSE Small Cap (April 1, 2003 - October 31, 2016), NSE Mid 100 (January 3, 2005 - October 31,2016), and NSE free float Small Cap 100 (January 3, 1999 - October 31, 2016) were explored.

In the analysis of temporal dependency in liquidity based portfolio returns, four portfolios were constructed, two each from both NSE and BSE. In this process, we considered the sample frame of constituent companies in NSE 500 and BSE 500 indices as on October 31, 2016. We excluded the companies which were not active in the respective markets in any period from January 1, 2010 and those with missing data. In this way, we chose 433 companies from BSE and 339 companies from NSE for the portfolio construction. The year 2010 was considered as the starting year in order to retain maximum number of companies in the portfolio construction whereby liquidity characteristic across the two extreme portfolios would remain less biased or less diluted. From this refined list, portfolios of highest liquidity and lowest liquidity were constructed for each month from January, 2010 upto October 2016 based on the monthly average of each company's daily trade volume for each market. Those 20 companies which had highest monthly trade volume constituted the highest liquidity portfolio and those 20 companies with lowest trade volume made up the lowest liquidity portfolio for the respective months. In the next step, daily returns of individual securities in each monthly portfolio were calculated for the respective months from their daily close prices and were averaged daily across the 20 securities to obtain the daily return of the respective portfolio. The daily returns of each of these portfolios : BSE Highest 20, BSE Lowest 20, NSE Highest 20, and NSE Lowest 20 were explored to examine the variations in dependency across market liquidity over the period from January 5, 2010 to October 31, 2016.

➷ **Description of Test Considered in this Study :** In this analysis, we resorted to detrended fluctuation analysis technique (DFA1) (Podobnik, Fu, Jagric, Grosse, & Stanley, 2006). In this method, first, the deviation from the arithmetic mean of time series x_i of total length N is taken, $y(k) = \Sigma^{k}_{i=1} (x_i - \bar{x})$. Then the random walk $y(k)$ is divided into boxes of equal length n and the local trend of $y(k)$ is calculated in each box by a least-square fit. The random walk $y(k)$ is detrended by subtracting the local trend $y_n(k)$ in each box, and for a given box size n, the root-mean-square deviation is calculated by :

$$F(n) = \sqrt{\frac{1}{n} \Sigma^{n}_{k=1} [y(k) - y_n(k)]^2}$$

The detrended fluctuation function, $F(n)$, would follow a scaling law $F(n) \propto n^{\alpha}$ if the time series x_i is power-law auto-correlated. The scaling exponent $\alpha > 0.5$ indicates time series with power-law correlations, while the $\alpha < 0.5$ indicates time series with power-law anti-correlations and $\alpha = 0.5$ corresponds to time series with no or only short-range auto-correlations ; 50 different window sizes ranging from 10 to 1000 were considered in the calculation of average fluctuation function of the respective windows in the analysis and linear local de-trending was done in the analysis. Since the analysis is carried out across pre-defined sub periods as well as for the entire data period, the possibility of erroneous conclusions based on scaling exponents due to structural breaks in the data is minimal.

Empirical Results and Discussion

(1) Dependence on Benchmark and Broad Market Indices' Returns and in Their Volatility : The Table 1 presents the Hurst exponents (*H*) values calculated through detrended fluctuation analysis (DFA1) method. The scaling exponents are the slope coefficients of the log of fluctuation function values (DFA1) regressed on their respective block window sizes. From the results, we learn that the scaling exponents calculated are above 0.5, which means, the presence of long memory process in all the indices across the periods of analysis. Broad market indices reported higher scaling exponents compared to benchmark indices. Overall, even though the long range dependence is present, it is found to be weak in all the cases of market returns of broad and benchmark indices. In the post crisis period, the persistence in the return increments is higher than that in the pre - crisis period. Broad market reported higher values of Hurst exponent than that of benchmark indices.

The Table 2 presents the results of examination of long memory process in the return volatility. Both the broad market and benchmark indices' squared returns were considered in the analysis. Contrary to the results of persistence in the return process, the analysis shows strong level of persistence in all the indices both in whole

Table 1. Long Memory in the Indian Equity Market : Scaling Exponents of Broad Market Indices

SENSEX (1990-2016)	NSENifty (1995-2016)	BSE500 (1999-2016)	NSE500 (1999- 2016)
Detrended Fluctuation Analysis (Entire Period)			
0.5395758*	0.5127295*	0.571986*	0.5544705*
Sub-Period 1 (Pre 2008)			
0.5313159*	0.5011315*	0.5612003*	0.5356833*
Sub-Period 2 (Post 2008)			
0.552542*	0.5516821*	0.5920444*	0.5846367*

Note. *indicate that the null hypothesis of no-long range dependence is rejected at the 1% level of significance. The values are calculated by the authors and data drawn from CMIE database.

Table 2. Long Memory in the Return Volatility of Indian Equity Market : Scaling Exponents of Broad Market Indices

SENSEX (1990 - 2016)	NSE Nifty (1995 - 2016)	BSE 500 (1999 - 2016)	NSE 500 (1999 - 2016)
Detrended Fluctuation Analysis (Entire period)			
0.7999422*	0.7401015*	0.7593542*	0.7727917*
Sub-Period 1 (Pre 2008)			
0.7882112*	0.6753977*	0.7244032*	0.7559595*
Sub-Period 2 (Post 2008)			
0.8433153*	0.8082076*	0.7971388*	0.7853919*

Note. *indicate that the null hypothesis of no-long range dependence is rejected at the 1% level of significance. The values are calculated by authors and data drawn from CMIE database.

period and also in the sub periods. It is observed that it is in the Sub Period 2, the persistence is found to be higher compared to Sub Period 1, especially in benchmark indices' volatility for which it is higher than that of broad market indices, opposite to what is observed in the index returns.

(2) Long Memory in Market Capitalization Based Indices' Returns and in Their Volatility : It is found in Table 3 that the value of scaling exponents declines across index returns of smallest to highest market capitalization. In other words, the long range dependence is higher among the low market cap index returns. The analysis shows that the lowest Hurst exponent value is for BSE Large Cap, while scaling exponent value increases as we move from mid cap indices to small cap indices in both the markets. Similarly, all indices also show positive power law correlations with the time span or the block size increases, indicating the persistence of autocorrelations in the return increments. In the examination of long memory in the return volatility, it is found that irrespective of the market capitalization category, all the indices evince higher level of positive persistence both across entire time period as well as in sub-periods unlike the case of returns. The Table 4 presents the results of the analysis. Unlike in the case of returns, the long range dependence is found to be stronger in a progressive order from small cap to high market cap indices in the third sub-period.

Table 3. Long Memory in the Indian Equity Market : Scaling Exponents of Market Cap Based Portfolios

BSE Mid Cap (2003- 2016)	BSE Small Cap (2003-2016)	NSE Mid Cap (2005 - 2016)	NSE Small Cap (2005- 2016)	BSE Large Cap (2005 - 2016)
Detrended Fluctuation Analysis (Entire Period)				
0.614714*	0.6294145*	0.5911719*	0.6171042*	0.5515671*
Sub-Period 1 (Pre 2008)				
0.5368054*	0.5540407*	0.522968^	0.4822588^	0.4691922^
Sub-Period 2 (Post 2008)				
0.6522544*	0.6728811*	0.620994*	0.6509966*	0.5620931*

Note. *indicate that the null hypothesis of no-long range dependence is rejected at 1% level of significance. Values calculated by authors and data drawn from CMIE database.

Table 4. Long Memory in the Return Volatility of Indian Equity Market : Scaling Exponents of Market Cap Based Portfolios

BSE Mid Cap (2003- 2016)	BSE Small Cap (2003-2016)	NSE Mid Cap (2005 - 2016)	NSE Small Cap (2005- 2016)	BSE Large Cap (2005 - 2016)
Detrended Fluctuation Analysis (Entire Period)				
0.727988*	0.7305737*	0.7453752*	0.7363239*	0.8336736*
Sub-Period 1 (Pre 2008)				
0.718671*	0.719247*	0.8452223*	0.8220444*	0.8888325*
Sub-Period 2 (Post 2008)				
0.714565*	0.7222177*	0.7094959*	0.7233099*	0.8275358*

Note. *indicate that the null hypothesis of no-long range dependence is rejected at the 1% level of significance. Values calculated by authors and data drawn from CMIE database.

(3) Long Memory in Liquidity Based Portfolio Returns and in Their Volatility : In the results given in Table 5, it is found that the scaling exponent for BSE highest 20 is close to 0.5, which corresponds with only short range auto-correlations unlike in the case of other portfolio returns. BSE indices returns evince lower persistence as compared to NSE portfolio returns. In both the markets, the values are lower for less liquid portfolios compared to

Table 5. Long Memory in the Indian Equity Market : Scaling Exponents of Market Liquidity Based Portfolios

BSE Lowest 20 (2010-2016)	BSE Highest 20 (2010-2016)	NSE Lowest 20 (2010-2016)	NSE Highest 20 (2010-2016)
Detrended Fluctuation Analysis (Entire period)			
0.5379*	0.5008*	0.5580*	0.5663*

Note. *indicate that the null hypothesis of no-long range dependence is rejected at 1% level of significance. Values calculated by authors and data drawn from CMIE database.

Table 6. Long Memory in the Return Volatility of Indian Equity Market : Scaling Exponents of Market Liquidity Based Portfolios

BSE Lowest 20 (2010-2016)	BSE Highest 20 (2010-2016)	NSE Lowest 20 (2010-2016)	NSE Highest 20 (2010-2016)
Detrended Fluctuation Analysis (Entire period)			
0.749255*	0.6535811*	0.5280514*	0.5328846*

Note. *indicate that the null hypothesis of no-long range dependence is rejected at 1% level of significance. Values calculated by authors and data drawn from CMIE database.

highly liquid portfolio returns of the same market. The examination in return volatility shows positive persistence as all the cases considered are weak. Especially, the dependence is found to be weak in case of portfolios from NSE while that of BSE show moderate as well as higher level of persistence. Opposite to the evidence in returns, BSE lowest 20 has higher level of persistence than BSE highest 20 ; whereas, NSE portfolios do not have significant difference between lowest and highest liquidity portfolio returns. The Table 6 presents the Hurst exponent values estimated in the return volatility of market liquidity based portfolios' returns.

Research Implications and Scope for Further Research

It is observed from this analysis that broad market index returns evince higher persistence compared to benchmark index returns, which surged in the post-crisis period. This trend reversed in case of volatility. The relevance of this information gains ground with significance of these indices in the market. The broad market indices BSE 500 and NSE 500 are the representation of the Indian equity market, respectively covering around 93% and 96% of the market capitalization of BSE and NSE. Benchmark indices BSE Sensex and NSE Nifty are comprised of most actively traded securities in the respective markets upon which various exchange traded funds, futures, and options, and other derivatives are traded. Therefore, it can be concluded that return formation in the Indian equity market shows long memory process, pricing of both underlying securities, market linked derivatives, and other financial products need to incorporate this characteristic of the market.

The analysis of index returns and volatility of market capitalization based indices brings to light the extent of the underlying source of this inefficiency in the market. The trade off between market cap and level of dependency was observed from the analysis. The level of persistence increased across the lower market cap indices' returns compared to larger cap indices, and the evidence was substantiated from both the markets. This observation can be attributed to the nature of the indices. Each of these indices reflects the size wise nature of the constituent securities. Finance literature portrays small-cap stocks as stocks with lower liquidity and highly affected by investors' behavioural traits and irrational sentiments in the market. The dependency can also be attributed to market micro-structure effects.

The results also show positive movement of market cap and level of dependence in case of the volatility of index returns. It substantiates the results observed in case of benchmark index volatility. If the same argument stated in the returns case is applied, both large cap and benchmark indices are expected to be constituted by securities which are more liquid and less prone to behavioural bias in the market. Having observed that all the indices evince long memory in returns, this evidence can be probably due to market micro-structure effects in the Indian market. The results of long memory in liquidity based portfolio returns and squared returns corroborate this conclusion. In case of portfolios from NSE, which have the highest turnover share in the Indian market, both raw returns and squared returns show higher level of long range dependence in the NSE highest 20 liquidity portfolio compared to that of NSE lowest 20 liquidity portfolio. However, in case of BSE portfolios, squared returns show stronger long range dependence in BSE lowest 20 compared to that of the BSE highest 20, though both were significantly higher.

Overall, the Indian equity market has evinced long memory in the price formation which is an indication of the possibility of slow information absorption of the market, which can be a reflection of behavioural factors or market micro-structural factors, which need further focused investigation in those lines. Similarly, the investor categories in the market also need to take into account the necessity of incorporating the long-range dependence characteristic into prices and also has to consider the information on nature of dependence found out in the portfolio return volatility in the pricing models of both common stocks and derivative securities.

Concluding Remarks and Limitations of the Study

The study observed long range dependence in the market returns and in their squared returns in Indian market, and it has also increased in the post-crisis period. The changes in the direction of dependence across indices indicate possible effects of behavioural factors and market microstructure in the Indian market. Therefore, pricing of both underlying securities and market linked derivatives and other financial products need to incorporate this characteristic of the market and efforts to bring more transparency and faster information dissemination in the market need to be addressed. One of the limitations of the study is that the examination focussed on the mono-fractal analysis of dependencies keeping aside the multi-fractal analysis.

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About the Authors

Saji George is a Ph.D. Research Scholar in the Department of Economics. He has formerly worked as a Lecturer with T. John Institute of Management and Science and holds a Master's degree and M.Phil degree in economics.

Dr. P. Srinivasa Suresh is an Associate Professor with North Eastern Hill Central University, Shillong teaching financial economics and statistics. He pursued M.A. and M.Phil. in economics from S.V. University, Tirupati, and Ph.D. from the Central University of Hyderabad, Andhra Pradesh, India.